LeviProps: Animating Levitated Optimized Fabric Structures using Holographic Acoustic Tweezers

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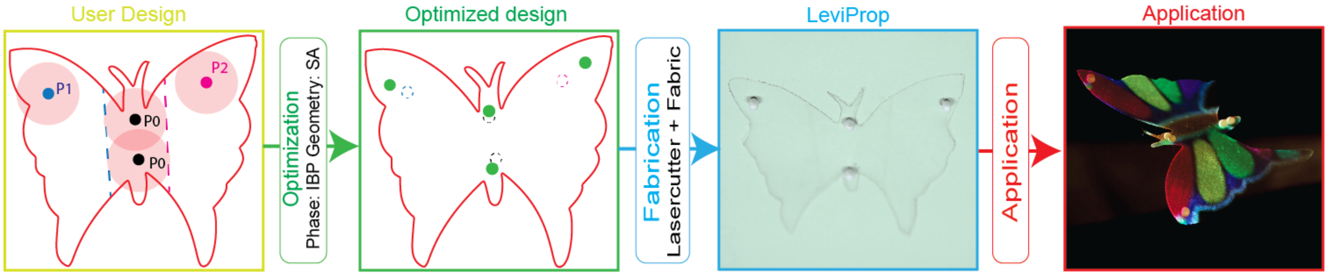

Figure 1. *LeviProp* provides a design method for creating levitated props combining a light piece of fabric with attached anchor beads: (a) We input the outline design and animation constraints (i.e. moving parts and rotations); (b) Our novel algorithm optimizes the location of the anchor beads on the fabric, obtaining maximum trapping forces on the structure (c) The final design is easy to build with a laser cutter; and (D) can be levitated in an interactive way.

ABSTRACT

LeviProps are tangible structures used to create interactive mid-air experiences. They are composed of an acoustically-transparent lightweight piece of fabric and attached beads that act as levitated anchors. This combination enables real-time 6 Degrees-of-Freedom control of levitated structures which are larger and more diverse than those possible with previous acoustic manipulation techniques. *LeviProps* can be used as free-form interactive elements and as projection surfaces. We developed an authoring tool to support the creation of *LeviProps*. Our tool considers the outline of the prop and the user constraints to compute the optimum locations for the anchors (i.e. maximizing trapping forces), increasing prop stability and maximum size. The tool produces a final *LeviProp* design which can be fabricated following a simple procedure. This paper explains and evaluates our approach and showcases example applications, such as interactive storytelling, games and mid-air displays.

Author Keywords

Levitation, design methods, tools, fabrication, mid-air UIs.

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INTRODUCTION

The idea of controlling matter to create advanced user interfaces has inspired HCI research, from concepts like the Ultimate display [30] to Radical Atoms [11]. Magnetophoresis [12] has been explored as a method for contactless control of matter. Ultrasonic levitation is another method [2,9,24,34] that has received significant attention for several reasons: a) no specific physical properties (e.g. ferromagnetic or dielectric) are required for the manipulated matter, allowing manipulation of materials ranging from polystyrene beads to coloured liquids [8,25], or even food [32]; b) it is low-cost compared to optical manipulation [14]; c) it is not harmful for the human health [3]; d) can reach tens of centimetres [6]; and e) it can manipulate multiple particles with fine control on the position [17]. However, apart from some exceptions which require high-power and can control only one particle [1,7,10,15], acoustic techniques are limited to small spherical particles (i.e. ~2mm) and shapes made of points, greatly limiting their expressiveness as interfaces.

This paper presents *LeviProps*, which are tangible levitated props created by combining lightweight and acoustically-transparent fabric (e.g. Super Organza) with attached polystyrene beads. The fabric provides a continuous and free-form 2D surface, adding to its expressiveness or even acting as optical diffusers for mid-air displays. The beads act as levitated anchors that support the fabric and enable dynamic control of the props. *LeviProps* can be manipulated in mid-air with up to 6 Degrees-of-freedom (DoF) and be composed of multiple moving parts called levitation primitives. A primitive is a set of one or more beads attached to the fabric that retain their relative position (i.e. move and rotate together). Primitives can be animated independently, e.g. the butterfly in Figure 1d is composed of 3 primitives:

the body primitive has 2 beads, and each wing primitive has 1 bead. This allows moving and orientating the butterfly with 6DoFs while the wings flap independently (we encourage the reader to check the supplementary video).

This paper explores the design space of *LeviProps*, analyzing the main factors that influence their performance and their capabilities. Particularly, we first evaluate the influence of the location of the anchors on the LeviProp overall trapping stiffness (i.e. converging forces allowing levitation), revealing that location is a critical factor.

Based on this finding, we propose a method to optimize the design of *LeviProps.* The method combines the Iterative Backprogation (IBP) algorithm [17] (i.e. state of the art method for multi-particle levitation); and, more critically, a simulated annealing (SA) search [31], to find the positions of the beads on the prop that result in maximum trapping force.

We then provide a numerical evaluation of the improvements on trapping stiffness achieved by our method, and an empirical exploration of the prop sizes that can be successfully levitated for different number of anchors.

We integrate our optimization method into an authoring tool (Figure 1a-c) to allow easy design and fabrication of *LeviProps*. This tool and the levitation setup used are available online[1] as an extension to Ultraino [16], an open solution for levitation. Thus, *LeviProps* can be used by other practitioners and is compatible with any levitator setups supported by Ultraino.

Finally, we present several examples, to illustrate the potential of *LeviProps* to create interactive experiences. Simple props can be used as visual physicalizations or small displays for interactive storytelling. Complex props can be used to create games, shape-changing displays and animated objects in mid-air, which all can be manipulated in real-time.

In summary, this paper includes the following contributions:

- A design and fabrication method to create levitated and dynamic surfaces of arbitrary shapes (*LeviProps*).
- A study of the influence of anchor locations on the overall trapping stiffness on the *LeviProp*.
- An algorithm that optimizes the position of the anchors on the prop, to maximize the resulting trapping stiffness according to its design and intended animations.
- A formative investigation of the amount of fabric that can be levitated depending on the numbers of anchors.
- Open-source tools to easily design and build *LeviProps*.
- Proof-of-concepts applications that demonstrate the capabilities of *LeviProps*.

RELATED WORK

Acoustic fields can exert a radiation force on the particles contained within [5]. This force can be used to paint or sculpt in fur [29] or sand and liquids [18]. Specific field shapes exert converging forces that can trap particles in mid-air [4]. For instance, a standing wave formed between an emitter and an opposite reflector will trap particles at its nodes [33].

Using arrays of emitters with controlled phase, the emitted field can be changed dynamically, allowing for instance to move particles in 3-D using four opposed arrays [24,25]. Later, two opposed arrays were used to create modular systems that can levitate 1 particle in 3D [26]. It is also possible to levitate particles using a single array that generates acoustic tractor beams [19].

Using multiple levitated particles to represent objects is an emerging research field [24]. For example, Floating Charts [27] is a modular display that levitates multiple particles for composing dynamic charts. JOLED [28] used levitated particles to form a grid of particles that act as a screen. Recent advances [17] (IBP- used by our approach), allow full individual control of multiple levitated particles, enabling mid-air 3D shapes (e.g. a cube) made of individual particles.

Levitation can also be created with the use of metamaterials [13,20,21] or a combination of them with phased arrays [23]. However, although this approach is simpler, in terms of electronics and cost, it only permits static levitation or very limited movement. Therefore, we focus our attention on displays relying on phased-arrays given their versatility and more widespread adoption, with solutions becoming available during the recent years (e.g. Ultrahaptics dev kit[2] and Ultraino [16]).

Being able to position particles in mid-air without the need of support enables a wide range of novel displays. However, as exposed in the previous subsection, most levitation techniques enable to move one particle or a group of them but not individually. Also, the particles are spherical and much smaller than the wavelength.

Some techniques have gone beyond these limitations. Particles larger than half-wavelength can be levitated close (a couple of millimetres) to a powerful sound emitter [1], but this could be also be obtained by traditional mechanical methods. The size limit can also be surpassed using special types of acoustic fields [10,15] but they require very high-amplitude and only one particle can be trapped. Non-spherical particles [7] can be stably trapped but the particles are still smaller than half-wavelength or require larger arrays and high-power [10]; also, only one particle at the same time is possible. Recently, the technique of Holographic Acoustic Tweezers [17] has shown to be able to trap and manipulate multiple particles individually; however, only up to 12 particles were stably levitated and the minimum separation between them (around 1 cm) only allows for coarse graphics.

[1] https://github.com/asiermarzo/Ultraino/leviprops

[2] https://www.ultrahaptics.com

Our approach, using particles as levitated anchors to manipulate continuous display surfaces goes beyond the limitations of previous approaches and enables more expressive mid-air displays. Our proposed algorithm is also the first one to optimize the location of the traps to maximize trapping stiffness (i.e. approaches such as [17,19] optimize the phase of the transducers, not the position of the traps). Finally, while the use of threads [27] or small patches of fabric [17] has been shown before, we are the first ones to provide a systematic exploration of this space, providing novel algorithms, reusable tools and methods.

LEVIPROP: APPROACH AND RELATED CHALLENGES

LeviProp is based on the creation of various levitation primitives, which can then be combined into single interactive prop with complex shapes and animations. Primitives are attached to a piece of fabric and can be decomposed in three types (see Figure 2): single beads (i.e. position control only); dual beads (i.e. 3D position and orientation around 2 axis); and 3 or more beads (i.e. 6DoF control). A single particle primitive is easier to fabricate and place on the levitator; however, richer manipulations require primitives involving more particles.

The ability of each primitive to support fabric and to be levitated depends on the total trapping force that can be exerted on its attached anchors. In turn, such stiffness depends on the number of anchors and on their location. Stiffness tends to decrease linearly with the number of traps [17]. Thus, even if the total force applied should be similar, the force of each trap will decrease if more anchors are used.

As our study reveals, the trapping forces on the anchors will change based on their 3D location, both within the primitive (local position) and within the levitator (global position). Our approach exploits this, providing the designer with the locations of the anchors that maximize the trapping forces for a given *LeviProp* and interactive experience. This involves two steps. First, we use the IBP algorithm [17] as an optimum algorithm for multipoint levitation (i.e. create a levitation trap for each anchor of the *LeviProp*). Second, we use a novel Simulated Annealing (SA) search to compute the optimum location of the anchors within the prop, given the shape (e.g. design, moving parts) and animations (e.g. translations/rotations) required by the designer.

The following subsections provide a background description for our approach (multipoint levitation algorithms, stiffness computation), followed by an exploration of the impact of anchor positions and numbers on the trapping stiffness of the props. Our novel SA algorithm is detailed later in the paper, once the need for position optimization has been illustrated.

Multipoint levitation algorithms: Holographic Acoustic Tweezers

Creating *LeviProps* involves the generation of one levitation trap for each anchor in the prop. The computation can be decomposed as the combination of a focusing pattern (determines the position of the traps) and a fixed levitation signature [19] (determine forces along each axis of the trap).

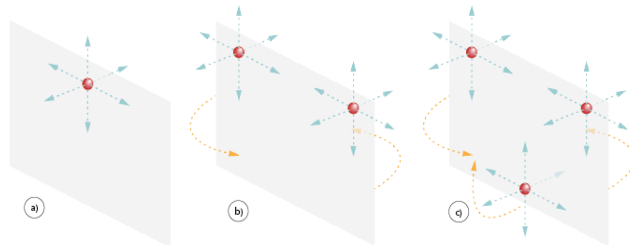

Figure 2. *LeviProp* primitives: (a) Single bead (3D position); (b) Dual bead (Position and yaw); (c) Multi-bead (6DoFs).

Let J be the set of traps to be created in the *LeviProp*, with each point $j \in J$ defined as a tuple j={ Φ_j, p_j}, with $\Phi j \in [0,2\pi)$ representing the phase of the point and $p_j \in \mathbb{R}^3$ its 3D position. Let T be the set of transducers in our levitator, with each transducer $t \in T$ again represented as a tuple t = { Φ_t, p_t} describing the transducers phase and position. A single point focusing pattern can be computed as:

$$H_j(t) = \frac{2 \cdot J_1(k \cdot r \cdot \sin(\theta))}{k \cdot r \cdot \sin(\theta)} \cdot \frac{P_{ref}}{\boldsymbol{d}(p_j, p_t)} \cdot e^{\left(\Phi_j - k \cdot \boldsymbol{d}(p_j, p_t)\right)i} \quad (1)$$

Here, transducers are modelled as a piston source [22] of radius r=5mm. P_{ref} represents the transducer's reference pressure at 1m distance ($P_{ref} = 0.17\ Pa$, in our case); k is the wavenumber ($k = \omega / c_0$) ; d is the distance between the transducer and the point; θ is the angle between the transducer´s normal and the point, and J_1 represents a Bessel function of the first kind.

The multi-point focusing patterns satisfying J can be computed using the Backpropagation (BP) method [17], by adding the single point patterns obtained from Eq (2):

$$H_T(t) = \sum_j^J H_j(t) \quad (2)$$

Such BP approach (also named as naïve) is simple to implement, but it can result in suboptimum trapping as illustrated later in the paper (Figure 3). We instead use the Iterative Backpropagation (IBP) algorithm [17], which iteratively refines the phase of the points in the *LeviProp* to improve trapping. At each iteration, the phase Φ_j of each focus point is updated as in Eq (3). The final focusing pattern to apply to the transducers is then computed by applying Eq (1) and (2) again (with the updated focus point phases).

$$\Phi_j = arg\left(\sum_t^T \frac{H_T(t)}{\|H_T(t)\|} \cdot e^{\left(k \cdot \boldsymbol{d}(p_j, p_t)\right)i}\right) \quad (3)$$

Finally, both methods require the use of a levitation signature. We add a phase offset of π radians to the transducers in the top array, which maximize vertical forces for each trap, required to levitate larger *LeviProp* sizes.

Acoustic forces generated and Trapping stiffness

Given our transducer's model, the complex acoustic pressure $\boldsymbol{P_t}(p)$ contributed by each transducer t at a given position p is computed as:

$$\boldsymbol{P_t}(p) = \frac{P_{ref}}{\boldsymbol{d}\ (p, p_t)} \cdot \frac{2 \cdot J_1(k \cdot r \cdot \sin\theta)}{k \cdot r \cdot \sin\theta} \cdot e^{i\,(\phi_t + k \cdot \boldsymbol{d}\ (p, p_t))}$$

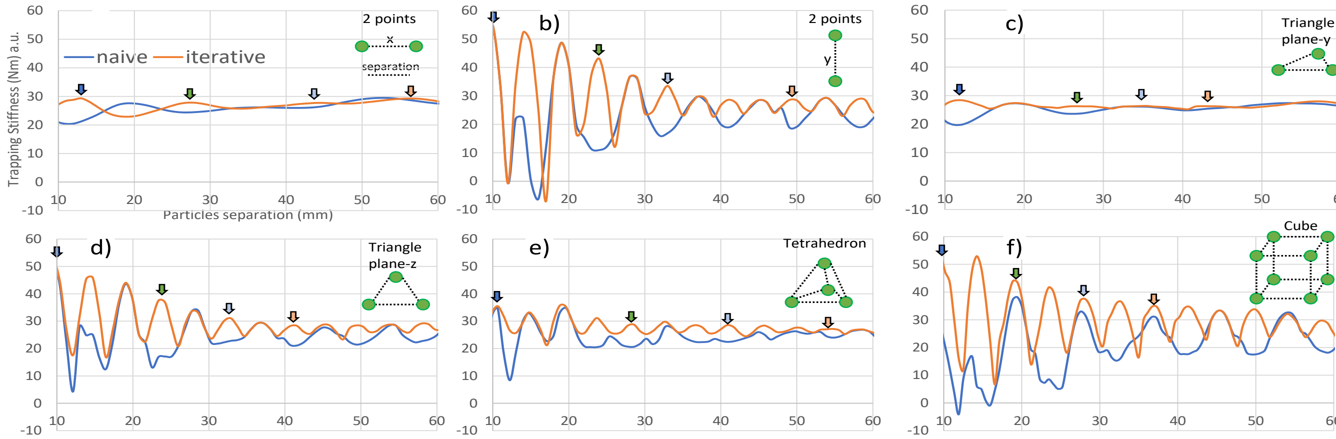

Figure 3. Changes in trapping stiffness for different bead structures as a function of their size. a) two beads with increasing horizontal separation. b) two beads with increasing vertical separation. c) a horizontal equilateral triangle of increasing side length; d) a vertical equilateral triangle of increasing side length. e) tetrahedron structure. f) cube structure. The blue line is the Naïve implementation (non-iterative backpropagation) whereas the orange line is the iterative backpropagation (IBP).

The total far field generated by our set of 512 transducers (two opposed arrays of 16x16 transducers) can be computed as the summation of the contribution of each individual transducer $\boldsymbol{P}(p) = \sum_t^T \boldsymbol{P_t}(p)$.

The acoustic forces exerted on spherical particles significantly smaller than the wavelength can then be computed by using the negative gradient of the Gor'kov potential ($\vec{\mathbf{F}} = -\vec{\nabla}\boldsymbol{U}$), which is defined as follows:

$$\boldsymbol{U}(p) = 2K_1(|\boldsymbol{P}|^2) - 2\,K_2\left(\left|\frac{\partial\,\boldsymbol{P}}{\partial\,\mathrm{x}}\right|^2 + \left|\frac{\partial\,\boldsymbol{P}}{\partial\,\mathrm{y}}\right|^2 + \left|\frac{\partial\,\boldsymbol{P}}{\partial\,\mathrm{z}}\right|^2\right)$$

$$K_1 = \frac{V}{4}\left(\frac{1}{c_0^2\rho_0} - \frac{1}{c_s^2\rho_s}\right) \quad ; \quad K_2 = \frac{3V}{4}\frac{\rho_0 - \rho_s}{\omega^2\rho_0(\rho_0 + 2\rho_s)}$$

Where $\boldsymbol{P}$ represents the complex pressure at that specific point $\boldsymbol{p}$. Variable $\boldsymbol{V}$ represents the volume of the spherical particle, $\boldsymbol{c}$ and $\boldsymbol{\rho}$ represent the speed of sound and density of each medium (subscript *0* refers to air, while subscript *s* refers to particle material).

As we described above, our levitation algorithms create levitation traps with converging forces in the x, y and z directions to retain the particle in place. Such converging forces can be computed and measured using the Laplacian of the Gor'kov potential ($\nabla^2\boldsymbol{U}$). The Laplacian is equivalent to the stiffness coefficient *(k)* of a damped spring model and allows us to measure the trapping strength on the beads of a *LeviProp* as the addition of the individual forces exerted on each of its beads:

$$Stiffness(J) \propto \sum_j^J \nabla^2\boldsymbol{U}\,(p) \qquad (4)$$

Local placement of the beads and trapping stiffness

The total trapping stiffness on the levitation primitive is influenced by the relative location of its anchors. Figure 3 illustrates the trapping stiffness inside our particular levitation system (described later in the paper), for primitives of 2, 3, 4 and 8 anchors, but varying the anchor positions. Please note that the case for a single bead primitive does not allow for position optimization and will not be discussed.

Figure 3a-b illustrates the case of two beads placed at the center of the array, showing the trapping stiffness that the levitator can exert on the anchors as the separation between them increases (1-5cm). Figure 3a illustrates the separation along the horizontal axis and Figure 3b along the vertical axis. Figures 3c-d illustrate the case of a 3-bead geometry, arranged as an equilateral triangle of increasing side (1-5cm), again arranged in a horizontal and vertical fashion (Figures 3c and 3d, respectively). Figure 3e shows the separation between beads of a tetrahedron geometry. Figure 3f illustrates a cube geometry made with 8 beads at the vertices.

In general, using the iterative algorithm (IBP) leads to stronger trapping forces in all cases when compared to the naive BP algorithm, justifying its inclusion in our algorithm.

More critical for our approach, Figure 3 also shows that the location of the beads can lead to a very significant difference on the trapping stiffness obtained on the whole structure. For the 2-bead case, Figures 3a-b reveal that the strength of the trapping varies very significantly with bead distance, particularly for the vertical case. This trend applies to all the shapes that have particles opposed to each other in the vertical direction (cube in Figure 3f, display in Figure 10e).

Thus, the decision of where to put the anchors in the fabric becomes a key factor for successful *LeviProps*. Changes in the distance between the anchors of just a few millimeters result in large variations in stiffness. This indicates that a relocation of the beads of just a few millimeters (i.e. minimum differences with the designer's initial shape) can transform a poor *LeviProp* (i.e. weak overall trapping forces) into an optimum *LeviProp*.

This is a key aspect of our approach, as it indicates that given an initial geometric design for the *LeviProp* (i.e. designer's selected locations), it should be possible to refine their positions, improving trapping even with minimum anchor displacements from the initial design. The next section will analyze if the improvements in such optimum geometries are retained, even after moving or rotating the *LeviProp*.

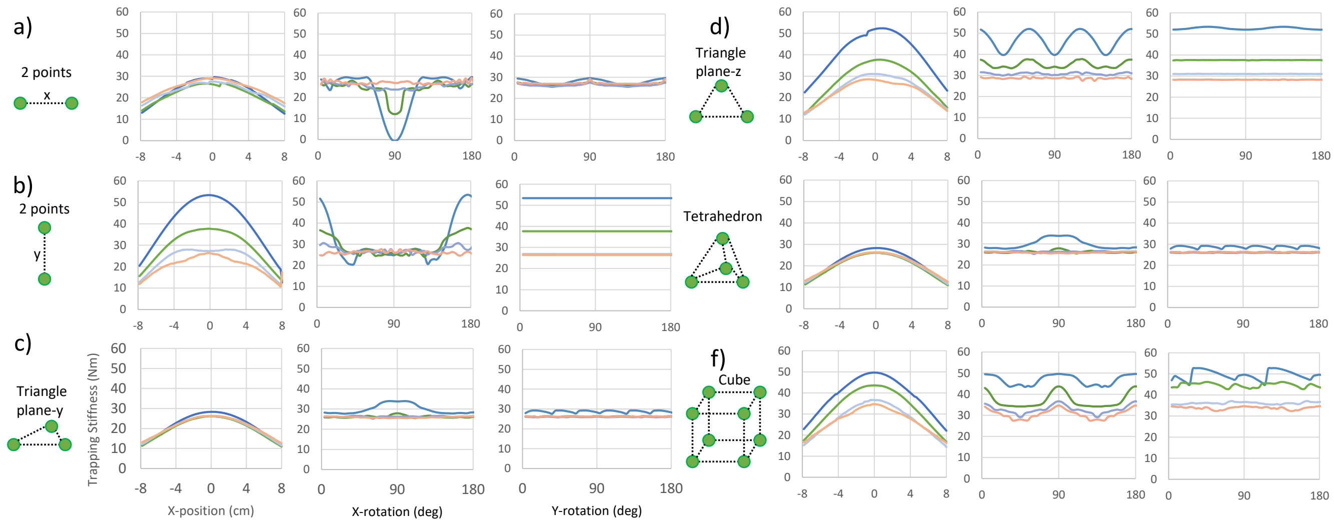

Figure 4. Changes in trapping stiffness for our selected optimums reference primitives, when translations and rotations are applied. The Figure shows that improvements in trapping quality remain even after transformations.

Effects of displacement and rotation of the prop

The previous simulations reveal that the placement of the anchors on the levitation primitive will have very significant effects on the obtained trapping quality, at least while the primitive remains at the simulated location (i.e. the center of the array). Here we explore if this optimized placement will remain adequate as we move/rotate the primitive inside the levitator.

To do so, we use the optimum geometries identified from the previous study, which were highlighted as colored arrows in Figure 3. We then simulate the trapping stiffness exerted on these selected geometries, as we move and rotate them inside the levitator, and we look for changes in their relative stiffness (i.e. if a geometry A had better stiffness than B in Figure 3, will stiffness still be better as we move/rotate it?).

As the geometries are translated across the levitator, the trapping quality follows a predictable pattern, being maximum at the central area of the levitator and linearly decaying as the primitive moves towards the edges of the levitator (similar trends were found for displacements along other axis). This is caused by the field decay of the transducers as the angle between them and the particle increases, i.e. particles in the center receive more contribution from all the transducers. However, relative stiffnesses between geometries are generally retained, and changes can only be found for cases in which the geometries had very similar stiffness values from the beginning. This allows us to not consider the effects of translations on the trapping quality of our *LeviProps*.

The effect of rotation varies depending on the axis of rotation of the levitated geometry. Given that our levitator is symmetric around the vertical (Y) axis, rotations along the Y axis show neglectable effects (very small changes in stiffness). In contrast, rotations around the X axis can show very significant changes in stiffness and hence cannot be ignored:

- **Horizontal primitives:** Improvements are retained but small primitives suffer a large deviation on the stiffness depending on the angle. In this case, designers need to be aware that *LeviProps* are limited in terms of rotation along the X-axis if they are of small size.
- **Vertical Primitives:** The vertical primitives are in the YX plane. In this case, rotation around the Z axis have some symmetry, which could result in Z rotations having smaller effects on the trapping stiffness.

DESIGNING STATIC PROPS FOR DYNAMIC LEVITATION

Our analysis shows that it would be possible to improve trapping stiffness by optimizing the position of the anchors/beads on the fabric. This can be done by just relocating the initial beads locations by millimetric adjustments, thus not affecting the initial design of the user but leading to a larger trapping stiffness.

While translations (in any axis) and rotations around Y can be ignored in this process (i.e. small effects on the trapping stiffness on the structures), rotations around X and Z axis must be considered if the user wants his or her *LeviProp* to rotate along those axes.

The following subsections provide a description of our proposed algorithm, which optimizes the positions of the anchors on a *LeviProp* composed of multiple moving primitives. The algorithm also considers if the designer wants rotations of the *LeviProp* along the X and Z axis (previously identified as relevant). We first formalize the definition of our *LeviProps* and then provide a description of the SA optimization algorithm.

Defining LeviProps: From the image into the levitator

Each primitive in a *LeviProp* is formalized as a tuple $P_i = \{\{p_{i,0}, .., p_{i,n}\}, \beta_i, M_i\}$, where each $p_{i,j}$ represents a bead position (in local coordinates to the *LeviPop*), β_i represents the rotation range allowed for the primitive (i.e. rotations of $\pm\beta_i$ radians around the local axis X). Finally, M_i is a

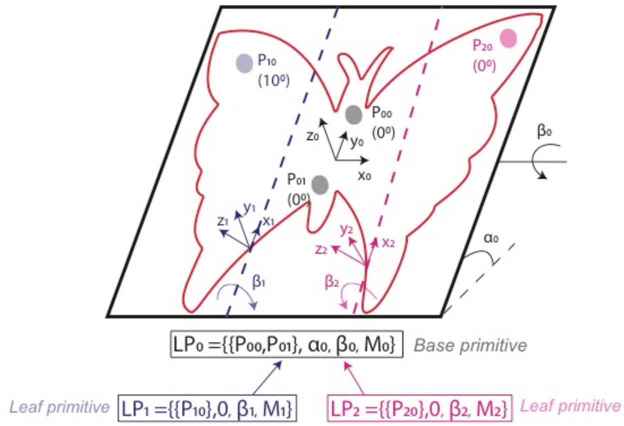

Figure 5. Example *LeviProp*, composed of 3 primitives. Top) Each primitive is defined in terms of its points $p_{i,j}$, rotation angle β_i and transformation matrix M_i. Bottom) primitives are arranged in a hierarchical structure to allow control of the whole LeviProp or their individual parts.

homogeneous transformation matrix, describing the position of the primitive. In the case of "leaf" primitives (i.e. LP_1, LP_2), this matrix describes the pivoting axis for rotations of the primitive. In the case of the base primitive ($i.e.\,\mathrm{LP}_0$), the matrix represents the initial location and orientation of the prop within the levitator. Please note that a *LeviProp* does not need to remain aligned with the levitator's axis (e.g. see initial inclination α_0, applied to the *LeviProp* in Figure 5).

The primitives in a *LeviProp* are structured in a hierarchical way (similar to a scene graph). Transformations to the base primitive LP_0 will affect all the points in the *LeviProp* (i.e. also beads in LP_1 and LP_2), but transformations on LP_1 or LP_2 will only affect their points. These potential transformations are defined by their rotation angles (β_i). For a given point $p_{0,j}$ in the base primitive and rotation angle $\beta \in [-\beta_i, \beta_i]$, its location in the levitator is computed as in Eq (5), where $R_X(\beta)$ represents a rotation matrix of β radians around X:

$$p = M_0 \cdot R_X(\beta) \cdot p_{0,j} \qquad (5)$$

Similarly, the position of a point $p_{l,j}$ in a leaf primitive (i.e. $l \neq 0$) with primitive rotation $\gamma \in [-\beta_l, \beta_l]$ is determined as:

$$p = M_0 \cdot R_X(\beta) \cdot M_l \cdot R_X(\gamma)\, p_{l,j} \qquad (6)$$

Transformation aware optimization

The formalization above, allows us to describe *LeviProps* composed of multiple individual parts that can be animated independently. These individual animations implicitly define the set of all the potential states of the *LeviProp*, i.e. potential locations of its beads, as per designers intended animations.

We sample the rotation ranges defined by each primitive (e.g. $[-\beta_i, \beta_i]$) at a rate of 3 degrees, producing a set of sampled angles B_i for each primitive. The Cartesian product of these individual sets $\mathrm{B} = \prod B_i$ represents the set of all potential rotations that can be applied to the *LeviProp* (e.g. $\mathrm{B} = \mathrm{B}_0 \times \mathrm{B}_1 \times \mathrm{B}_2$ for the example in Figure 5).

For a given set of initial positions of the points in the *LeviProp* ($J = \{p_{i,j}, \forall \mathrm{i}\}$), and in combination with Eq (5) and (6), B can be used to produce a final set *S(J)* containing all possible *states*. That is, each *state* in *S(J)* describes a set of possible positions of the beads in the *LeviProp*, for a given combination of the potential rotations defined by B.

Trapping stiffness optimization: Simulated Annealing

We use a Simulated Annealing (SA) approach to find the set of bead locations that provides maximum trapping stiffness across all the potential permutations applicable to the *LeviProp* (i.e. all *states* in *S*). For a given arrangement of the points, Eq (7) defines a cost function representing the quality of such arrangement, computed as the summation of trapping stiffness across all states in S:

$$Cost(J) = \sum_{s}^{S} Stiffness(s) \qquad (7)$$

The designer's initial placement of the anchors determines the initial state x_0. Neighbor states x_i are obtained by random permutation from the prior state, retaining the maximum displacement allowed per point. The transition acceptance between states follows the traditional acceptance criteria by Kirkpatrick [31]. The initial temperature was set to T=100 (adjustable in the tool). Each iteration's best solution (x_{best}) is accepted if the cost in the neighboring state is $x_1 > x_{best}$.

CHARACTERIZING LEVIPROPS

In this section, we provide an empirical exploration on the size of the levitation primitives that can be used to build *LeviProps*. We first describe our experimental setup (levitator) and the properties of the fabric used (Super Organza). We continue with an exploration of the maximum size for the primitives that can be levitated using our setup and methods. It is worth noting that while the results depend on our choice of levitation setup and fabric, both our software and hardware tools are open source, allowing reproduction by practitioners using other setups.

Experimental setup:

The system has two opposed arrays of 16×16 transducers controlled by an FPGA, an OptiTrack tracking system and a computer. The design of the arrays is a reproduction of the setup in [17], using Murata MA40S4S transducers (1cm diameter, operating at 40 kHz) and an Altera Cyclone IV EP4CE6 FPGA to receive phase updates from the computer and a UART protocol for communication (250 kbauds, ~90 updates per second). An OptiTrack Duo V120 system is used for tracking and correcting bead locations in real time. We made our design compatible with the Ultraino open source platform [16] (XML description files can be found in Supplementary Material), allowing reproduction of our results with either our same setup or other levitation arrangements supported by the platform.

Super Organza: Relevant properties

Super Organza is a thin sheer fabric traditionally made from silk. We used a microphone (B&K 4138 A-015 model) to measure the transmissivity of the fabric, observing decays in SPL of ~5% (i.e. from 141.6 SPL to 140.8 SPL, for a microphone placed 5cm in front of one transducer). This and

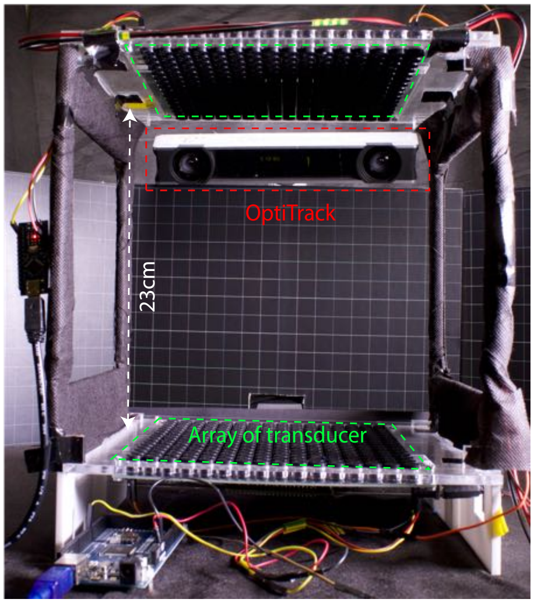

Figure 6: Overview of the components in our setup. We used two opposed arrays of transducers separated by 23 cm and an OptiTrack system to track the position of beads in real time.

its low density (5gr per square meter), makes Super Organza an excellent choice to implement *LeviProps*. Finally, Super Organza[3] is available in a range of colors, adding variety to the props that can be created.

Maximum size of levitation primitives

We tested the maximum size of fabric that can be levitated, according to the number of beads on the primitive. More specifically, we tested 7 different sizes (sides from 4-10 cm in steps of 1cm), attached to 1, 2, 3 or 4 beads, for a total of 28 primitives. In all cases, the bead locations were optimized using the approach described earlier, and the *LeviProp* was placed horizontally at the center of the levitator.

For each of our 28 primitives, we first placed individual beads (not attached to fabric) at the locations where the anchor beads of the *LeviProp* should be located (OptiTrack was used to automatically relocate them, in case that the bead had been placed at a secondary trap, similarly to [2]). These individual beads provided a visual reference to assess where the actual *LeviProp* should be placed. In the second step, we placed the *LeviProp* primitives on top on the reference beads matching them. This procedure was repeated 5 times per primitive (i.e. 28 primitives x 5 trials = 140 trials), and the trial was considered successful if the levitator could hold the primitive in place for 5 seconds.

Figure 7 shows the results obtained from this study, illustrating the success rate for each size and type of primitive. For the single-bead primitive case, we achieved successful levitation of props of up to 81 cm^2 (i.e. 9×9cm, see Figure 10a), with success rates above 80% for up to 49 cm^2. Maximum prop size decreases with the number of

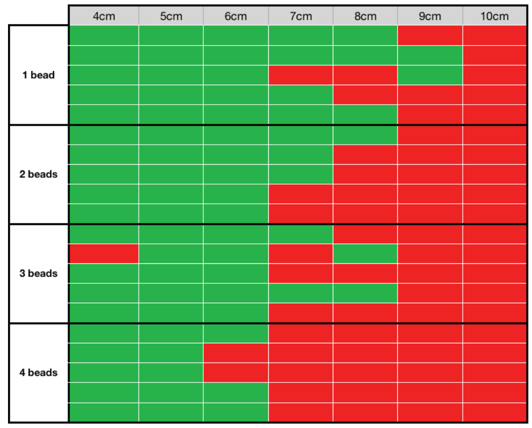

Figure 7. Results of testing 7 fabric sizes (4, 5, 6, 7, 8, 9, 10 cm) with 4 number of traps (1, 2, 3 and 4) separated by 2.5cm. Each primitive was tested 5 times, green is success and red is failure.

beads, and 100% success rate was obtained with sizes of up to 36 cm^2, for 2 and 3 beads. In the case of four-bead primitives, 100% success rate was obtained with props of 25 cm^2. It is worth noting that for 3 beads and a primitive of size 4 cm, levitation was not possible, as the horizontal distance between the particles was too small.

As expected, the size of the levitated props decreases with the number of beads. Even if their location is optimized, the solution reached still deviated from the ideal case where the overall trapping force would be the same than that of a single particle. However, the force of each trap decreases with the number of anchors, and failures were usually the result of one trap failing to hold its bead. This led to the primitive being held by a lower number of anchors (i.e. less holding force) and causing the remaining traps to progressively fail. Finally, small errors when attaching the beads to their exact location on the *LeviProp* also reduce trapping forces.

DESIGNING AND FABRICATING LEVIPROPS

This section describes an open source authoring tool to support the design of optimized *LeviProps*, as well as a simple fabrication method. This process involves multiple steps, from the definition of the *LeviProp* structure (e.g. primitives or desired rotation angles) and the optimization of the initial bead locations obtained by simulated annealing, to the actual physical fabrication. We use the butterfly example (see Figure 1 and 5) to illustrate the creation of a *LeviProp* that can rotate around the center of the levitator and flap its wings (i.e. one primitive per wing) by 10 degrees (Figure 8).

Designing *LeviProps*

Our tool provides support for the steps involved in the creation of *LeviProps*, as illustrated in Figure 8.

The designer starts by loading the outline of the desired *LeviProp* shape (Figure 8a). Next, the designer can select the number of anchors required to hold the "base primitive" (i.e. see the two black dots on the central body of the butterfly, in

[3] http://amaikegroup.com/fabrics/index.html

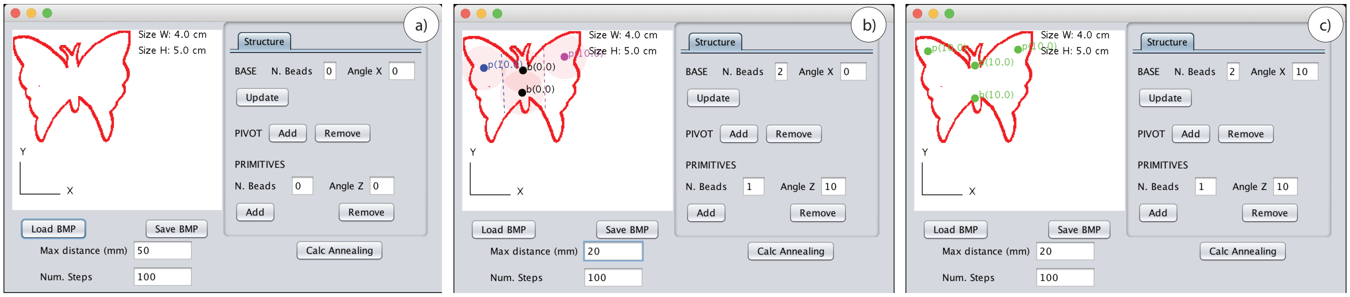

Figure 8. Steps followed by a designer using our authoring tool to design a *LeviProp*. The user loads the outline of the *LeviProp* (a); Primitives, relocation range and rotation angles are defined using the GUI (b); The software produces the optimized design based on the user constraints (c).

Figure 8b), dragging them to the approximate location where they should be placed (initial guess used as state x_0). The designer must also select the maximum distance within which these anchors can be relocated (i.e. red circles around black dots). This would be enough for the creation (and optimization) of simple props with no moving parts.

In order to add "leaf primitives", the designer must first select the pivoting point around which the primitive can rotate, by selecting two points on the image. This pivoting axis is indicated as a colored dashed line on the design (see blue and purple lines to the left and right of the butterfly body in Figure 8b). Finally, the designer selects the number of anchors required for each primitive and provides their estimated initial location, as well as the maximum relocation distance allowed. Each anchor is colored according to the primitive that it is associated with (matching the colour of its pivoting axis, as shown in Figure 8b), and relocation distances are again shown as circles.

Finally, the designer must also decide on the initial inclination applied to the LeviProp (see Angle X, in Figure 8a), as well as the rotation angles applied to each primitive around their pivoting axis (see Angle Z).

Once the design of the LeviProp and animation constraints (i.e. rotations) are defined, the system computes the optimized location for the beads, using our proposed SA method (see Figure 8c). The tool then produces a final image file including the shape of the outline, marks for the location of each anchor, as well as dashed lines to facilitate the rotation of the fabric around the intended pivoting axis.

Fabricating *LeviProps*

As a result of the previous steps, the tool produces an image file which is ready to be fabricated. The image includes the outline of the *LeviProp*, dashed lines at the position of each pivoting axis (i.e. to facilitate folding), as well as little marks for the location of each anchor bead.

This design can be fabricated with a laser cutter, following the steps illustrated in Figure 9. We first place the Super Organza fabric between two sheets of A4 paper (80gr). This avoids burning the fabric, as well as preventing it to be blown away under the effects of the laser cutter's fans. In our case, we adjusted our laser cutter settings (Universal Laser System: VLS2.30) to use paper with 0.01cm of thickness.

Once the cutting is finished, the beads can be glued onto the fabric at the designated points. We use an off-the-self spray glue to fix the beads on to the fabric. To avoid excessive glue and weight, we apply a small amount of glue onto a piece of paper. We then dip each bead onto the glue and place it at the designated point on the fabric.

APPLICATIONS

We illustrate the potential of LeviProps using five example applications (see Figure 10), each with varying numbers of anchors, and making use of the different possibilities provided by our method, like the use of one or more *LeviProps*, or *LeviProps* made of several primitives.

We also show a histogram along each image, showing the trapping stiffness of all the potential configurations explored by our optimization method. The optimum solutions identified by our algorithm are always shown at the bottom of each histogram (as the best solution found). The histograms also highlight the mean trapping stiffness of all these configurations (generally much lower than our optimum solution), as a way to provide an estimated value for the stiffness that would be most likely achieved if the location of the beads was selected randomly (i.e. not using our algorithm). Please note that Figure 10a was included to show the largest *LeviProp* we could levitate to date (9x9 cm), but a histogram cannot be provided as single-particle props cannot be optimized.

Figure 10b shows an example of a simple *LeviProp* designed with two anchor beads, which can be used as tangible notifications or reminders (i.e. similar to a conventional post-it notes). Such notifications could be combined with dynamic displacements of the *LeviProp* (i.e. move up and down and/or rotate, to attract the user's attention of the proximity of an event). It must be noted that the optimization of the

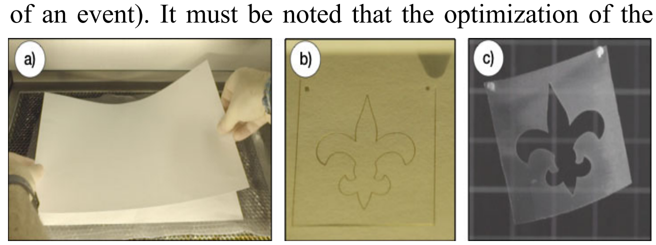

Figure 9. Fabricating *LeviProps*. a) Placing the SuperOrganza fabric between two sheets of papers. b) laser cutter cutting the outline of *LeviProp* and marks for the bead location. c) Final leviProp, produced by gluing the beads to the designated points.

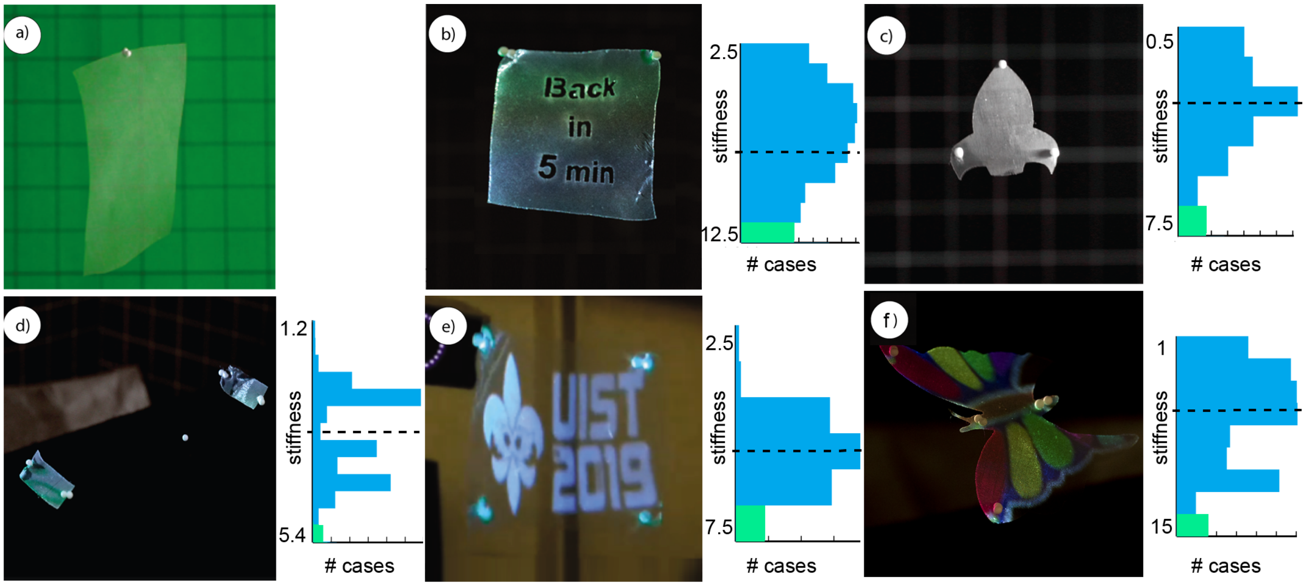

Figure 10. Example applications: Exploring large spaces (a). Permanent context data visualization in mid-air interface (b). Storytelling: Outline shape manipulated by three beads (c). Pong game, the users control each paddle independently (d). Animation: props can manipulate areas of the fabric by moving its anchors (e). Mid-Air display with projection mapping (f). Histograms of the stiffness trapping during the optimization is shown at the sides, black dash line is the mean, bottom is the optimum.

bead locations is independent of the actual shape (outline and or content/text inside the prop), making it possible to create a range of compatible *LeviProps*, each with specific meanings, simply by ensuring that the location of the anchors and overall surface of each one of them are equivalent. The low number of traps ensures high trapping forces for each anchor, making it easy to place/remove such notifications. Our optimization results in this case (translation only, 180º rotations) show trapping stiffnesses that are twice the ones that could be expected from randomly selecting the location of the beads (i.e. optimized result vs mean result) and its computation required 12’18’’.

Figure 10c shows an example of a *LeviProp* with 3 beads, where the shape can be related to rich/relevant meaning applicable for instance to interactive storytelling. Our example rocket rotates up to 90º (from horizontal to vertical) and supports vertical displacements, to illustrate that it is taking off. Again, trapping forces are large enough as to allow easy manual placement of the prop. Optimization resulted in trapping forces 2.5 times larger than the expected random case (i.e. mean), and computation required up to 61’33’’.

Figure 10d shows the case of several individual *LeviProps*, combined to create an interactive game (i.e. our rendition to the classic Pong). Particularly, we created two *LeviProps* (each one representing a paddle), and an independent bead to act as the ball. Even if using a higher number of particles, their distance allows for high trapping forces for each paddle (i.e. IBP optimizes phases as for each half of the array to focus on the closest paddle) and manual placement is possible. Improvements in trapping force are ~2.8 larger than a random choice, and computation required 6’6’’.

Figures 10e and 10f show cases where the surface of the *LeviProp* can be combined with dynamic projections. The first case (Figure 10e) illustrates the case of a mid-air display, allowing free control in 6DOF and rich projected contents (e.g. videos), with applications in multiple use cases, such as video conference applications where, besides receiving the video feed from our interlocutor, its local embodiment (the *LeviProp*) can freely move, as to better orient itself towards the user or to reflect its actions. However, optimization was computed as to only allow rotation around the horizontal axis and up/down movements (manipulations that, as per our tests, do not affect overall stiffness). In this case, an optimum solution is easy to reach (i.e. simply spacing the beads in each column to be placed within a standing wave), trapping improvements are smaller and computation required 12’18’’.

The final case (Figure 10f), shows the butterfly previously used for our examples, featuring a rich and meaningful shape, several movable primitives (i.e. flapping wings) and dynamic projection. This demonstrates mid-air display formats that are not constrained to rectangular shapes, and richer animations (i.e. rotations around the centre of the levitator and variable inclinations). The inclusion of animated primitives, however, results in a much more computationally intensive case (i.e. each potential position of the butterfly body and wings is tested, for each case explored by the SA method), resulting in an overall computation time of 1h:40’27’’, but also in very significant gains in trapping stiffness (peak vs mean trapping stiffness).

LIMITATIONS AND FUTURE WORK

Our exploration has demonstrated the feasibility of using *LeviProps* (both our methods and tools) to create structures that can be manipulated in mid-air, as well as a varied range of exciting interactive possibilities. The results obtained also illustrate the advantages of the optimization proposed in this paper. First, the required relocations of the particles are very small (1-2 mm), introducing minimal changes to the designers' initial choice of locations, and giving the designer the control on which particles can be relocated, and by how much. Second, our example applications also showed that the trapping stiffness obtained for such optimum solutions is greatly superior to the ones that could be expected from randomly chosen locations, providing double (or even more) trapping force than the mean cases. Also, the bottom parts of the histograms in Figure 10 (containing the best potential configurations) tend to show narrow tails, indicating that the probability of finding such optimized arrangements is very low, and further highlights the relevance of our method.

We believe that these methods, our open-source authoring tool, compatibility with open-hardware platforms (i.e. Arduino) and our simple fabrication methods will provide an exciting opportunity for others to continue to explore the potential of our concept. Beyond the current community of Ultraino users, we are planning to conduct a workshop with artists, both to explore more deeply the creative implications and potential for interaction with *LeviProps*, but also as an attempt to enrich the current community with new creative perspectives.

LeviProps however are not free of limitations. The multi-point levitation approach and setup that we used has demonstrated controlled and stable levitation of up to 12 particles but, in our experience, the additional weigh of the fabric reduces this number, e.g. we only managed to levitate 2 *LeviProps* of 4 particles each, or 4 props of two particles.

Our algorithm does not consider the weight distribution of the fabric along the geometry. This could be included to account for anchors holding extra weight or, what is more relevant from our experience, extra drag create when the anchor is moved/rotated. Thus, the inclusion of the fabric affects the maximum displacement speed of our particles and we tend to limit our animations to 5 cm/s and 20 deg/s, which are slower than the 70 cm/s allowed by other systems [24].

The fabrication process is simple but requires some practice to be mastered. Particularly, minimum amounts of glue should be used, as glue can easily erode EPS and add extra weight. Small errors when placing the beads can result in a final geometrical configuration that differs (even if minimally) from our tools' output. Hence, given the low costs and simplicity of our fabrication process, creating several copies of the intended *LeviProp* is recommended, for selecting the best later on.

Once fabricated, *LeviProps* are surprisingly resilient, beads stick firmly to the fabric and they can be extensively manipulated and reused. However, users should avoid directly stretching the fabric, as this changes the relative location of the beads (i.e. pulling on the edges of a square *LeviProp* will deform it into a rectangle, and the fabric will retain such shape).

CONCLUSION

We have presented *LeviProps*, a method that allows to acoustically levitate lightweight and acoustically-transparent structures that are larger than half-wavelength, arbitrarily shaped and continuous (i.e. as opposed of being made of points). Our approach relies on multiple beads attached to a lightweight fabric. The beads are trapped using multi-point levitation methods and used as anchors to manipulate the fabric. *LeviProps* created this way, can be moved with up to 6DOF and animated (i.e. a butterfly flapping its wings). We showed that the position of the beads on the structure significantly affects the total trapping stiffness. We introduced an algorithm to optimize the position of the beads on a given outline of fabric and created an authoring tool that helps designers to create their optimized *LeviProps*.

Firstly, we studied the trapping stiffness exerted on multiple traps depending on their geometrical arrangement (i.e. relative translation and rotation) and the algorithm used for calculating the traps (i.e. naïve or iterative). Secondly, we performed a formative experiment to investigate the largest piece of fabric that can be levitated depending on the number of beads and their geometry. In the first experiment, we found that while the iterative (IBP) algorithm improved trapping, the position of the anchors on the prop had a great impact on the final trapping stiffness, especially for structures with particles aligned along the vertical axis. In the second experiment, we found that the largest size of fabric which can be levitated with one particle is 81 cm^2.

To facilitate the creation of *LeviProps*, we have created an authoring tool that enables designers to build levitated tangible continuous surfaces. Additionally, the tool optimizes the initial position of the beads according to the designers' necessities and provides an optimum *LeviProp* that can be trapped with more stiffness. An easy-to-use method is presented for cutting the shape of the fabric with a laser cutter and gluing beads at the marked location.

Based on the results, the tool, and the method, we then designed proof-of-concept applications that exemplify several features such as different fabric sizes, number of beads, displays and animations.

We hope that this work will encourage designers to create levitated displays using *LeviProps*, exploring larger and more complex levitated structures and enabling richer mid-air displays and novel interactive experiences.

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